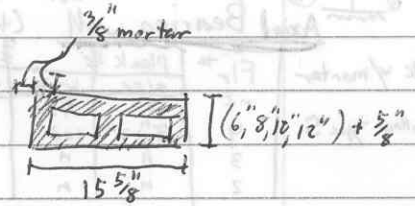


Masonry

CMU weight: light: $w < 105 \text{ pcf}$ low strength
 medium: $105 < w < 125$ ↓
 Norm: $125 \text{ pcf} < w$ high strength



shrinkage: drying shrinkage allow = 0.065%

Brick weathering: $C = \% \text{ water in cold water}$
 $B = \% \text{ water in cold + boiling water}$
 $\frac{C}{B} \geq 1.0$ is bad (No F/T ability)

	normal weathering		moderate weathering		special weathering	
	C	C/B	C	C/B	C	C/B
C62	-	-	22%	.88	12%	.77
C216	-	-	22%	.88	12%	.77

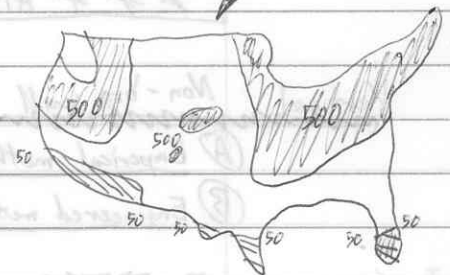
Vert Earth none	W I		
	<50	50-500	>500
horizontal Earth none	MW	SW	SW
horizontal Earth none	MW	SW	SW
horizontal Earth none	SW	SW	SW
horizontal Earth none	MW	SW	SW

Weather index = $\frac{\# \text{ of inches of rain}}{\text{year}}$ (# of F/T cycles)

IRA = internal rate of absorption = $(W_1 - W) \frac{30 \text{ in}^2}{A_{net}} < 30 \text{ grams}$

W_1 = brick weight @ 1 min in $\frac{1}{8}$ " water W = weight of dry brick

Mortar PC:L:S
 M 1:0:3 } req'd below grade } req'd on exterior
 S 1:1/2:4.5 }
 N 1:1:6 }
 O 1:2:9 }



SEE HW #2

Types PC:L:S, mortar cement

	PC	Fine ag	course ag (< 3/8")	lime
Grout Fine grout	1	3		0.1
coarse grout	1	3	2	0.1

low lift: grout @ each level

high lift: grout @ each story

Reinf

#4-6 typ. used grade 60 ($F_y = 60 \text{ ksi}$)



Prism strength

$f'_m = \frac{\text{failure load}}{\text{net area}}$ (correction factor) 1500 psi typ

Allowable comp. strength

$\frac{A_n}{A_g} \approx 0.4$ table $\frac{P}{A_{net}} = 2000 \text{ psi} = \frac{P}{0.4 A_g} \Rightarrow \frac{P}{A_g} = 0.4(2000) = 800 \text{ psi}$ block
 if want 1000 psi block $\frac{P}{A_g} = 1000 \text{ psi} = \frac{P}{(\frac{A_n}{.4})} \Rightarrow \frac{P}{A_n} = \frac{1000}{.4} = 2500 \text{ psi}$

A see empirical design for brg walls $\frac{h}{t} \leq 18$



check compressive stresses in block
 $F_c = \frac{P}{A} \leq F_{c,allow}$

B Engineered Method wall $h = 10'$
 Axial Bearing wall (trib area) = $10'$ (DL+LL) trib

block w/ mortar	Fir #	Plank #	size	weight	Floor D	DL tot	LL tot	total (plf) load on wall	wall load above	wall weight	total wall load	total wall stress	allowable comp. stress
8" hollow type S	rf	8"	75	15	40	30	1200 plf	0 plf	355 plf	1555	$\frac{1555}{A} < 65 \text{ psi}$	< 65 psi	handout
	4	8"x2"	100	20	120	40	1600 plf						
	3	"	"	"	"	"	"						
	2	"	"	"	"	"	"						

Area Table 6 p. A-4
 8" hollow = 35.5 plf/ft
 8" spaced = 85.8 plf/ft (\approx Table 3 p. A-3)
 12" hollow = 44.5 plf/ft

Table 5.4.2

* all mortar is PL-L mortar



$\frac{P}{A_{net}} = 2000 \text{ psi} \Rightarrow \frac{P}{A_g} = 0.4 (2000) = 800 \text{ psi}$
 Block type $\frac{A_n}{A_g} \approx 40\%$

Control Joints: A Empirical method B Engineered method

- spacing: 25' or 1.5h (smaller)
- @ pt. of weakness
- 1/2 spacing @ corner
- provide $A_{s,min} = 0.25 \frac{\text{in}^2}{\text{ft height}}$ as hor. reinf

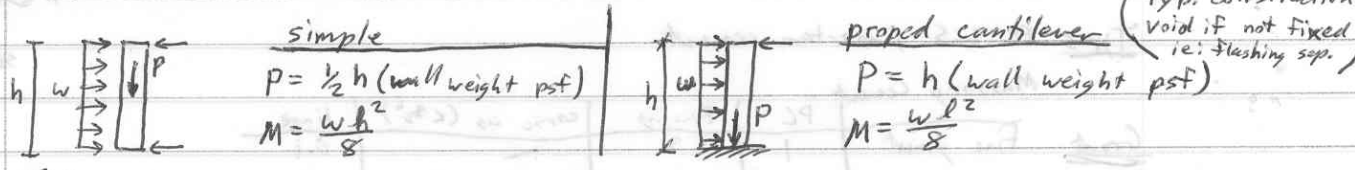
Find CCC: $CCC = \frac{0.00065}{2} + 0.00025 + 0.000055(5T_b)$
 $CCC \leq 0.001$ $\frac{0.001}{CCC} < 2.0015$

$l = 25'$ } min $l = 20'$ } min
 $l = 2.5h$ } $l = 2.0h$ }

* * * Alternate: if $A_{s,min} \geq (0.002 \cdot A_{wall,net})$ no control joint req'd (usually in seismic D, E, F)

Vertical Non-brg wall (wind on ext. walls)

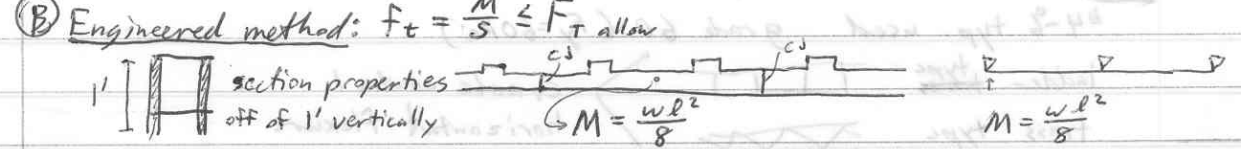
A Empirical method: see empirical design for non-brg walls $\frac{h}{t} \leq 18$
 B Engineered method: $f_t \text{ net} = \frac{M}{S} - \frac{P}{A} \leq F_{T,allow}$ (P = dead load only)



* for S, A see appendix A table 2

horizontal Non-brg wall (wind on ext walls)

A Empirical method: $\frac{l}{t} \leq \dots$ see empirical design for non-brg walls $\frac{l}{t} \leq 18$
 watch out for control joints: 25' } min



rein f. @ 8" increments

grade 60 $F_s = 24000$ psi allow

$F_m = \frac{1}{3} F'_m$ allow

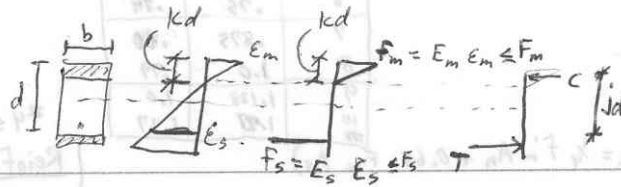
$F_b = \frac{1}{3} F'_m$

$E_m = 900 F'_m$

$n = 21.5$ ← for $F'_m = 1500$ psi

$E_{brick} = 750 F'_m$

$E_s = 29000000$ psi



Non-brg wall

Flexure

$M_s = A_s F_s j d$

$(M_{max}: f_s = F_s)$

$M_m = \frac{1}{2} b j k d^2 F'_m$

$(M_{max}: f_m = F'_m)$

$T = A_s F_s$

$C = \frac{1}{2} b (k d) F'_m$

solid, or assumed solid

$k = \sqrt{(pn) + \sqrt{(pn)^2 + 2(pn)}}$

rect. solid $j = 1 - \frac{k}{3}$

for 8" CMU $d = 3.81$ "

$n = \frac{E_s}{E_m} = \frac{29 \times 10^6 \text{ psi}}{900(1500 \text{ psi})} = 21.5$

$f_s = \frac{M}{A_s j d}$ given b, d, A_s, M

$F'_m = \frac{2M}{b j k d^2}$ find j, k

check $k d \leq t_{face}$ if ok, Face shell bedded only needed (can assume fully grouted)

$t_{face} = 1.25$ " for 8" CMU

Comp $\left\{ \begin{array}{l} \text{bar spacing} \\ \text{beff} = 6 \times \text{wall thickness} \\ \text{min} = 72" \end{array} \right.$

bar spacing # @ 24"	$A_s (\frac{in^2}{ft})$	pn	k	j	kd	M_s	M_m	over reinf ($M_m < M_s$)	under reinf ($M_s < M_m$)
					$< t_f$				
					$< t_f$				
					$> t_f$?

Partially grouted ($k d > t_{face}$)

T-beam flexure (needs either running bond or hor. bond bar $< 48\%$)

long approach (solve for $x_r f_m = x_c f_m$)

try $k d > t_f$; $K = \frac{k d}{d}$

$f_{m2} = \left(\frac{k d - t_f}{k d} \right) F'_m$ leave as variable

$f_s = n \left(\frac{1-k}{k} \right) F'_m$

$C_f = \frac{1}{2} (f_{m1} + f_{m2}) (t_f) \left(\frac{12"}{ft} \right)$

$C_w = \frac{1}{2} b_w (k d - t_f) F'_m$

$T = A_s F_s$

$C = T$

$M = A_s f_s j d$

(interpolate between 2 tries to get $k d$)

$C_f + C_w = T$

$(x_c) f_m = (x_r) F'_m$

if $(x_c) \neq (x_r)$ & if $(x_c) > (x_r)$ $k d$ moves up (closer to t_f)

Short approach (ignore web contribution (non-brg only))

(Flexure only)

$f_{m2} = \left(1 - \frac{t_f}{k d} \right) F'_m$

$f_s = n \left(\frac{1-k}{k} \right) F'_m \leq 24000$ psi

$p' = \frac{A_s}{b_w d}$

$k = \frac{(p' n) + \frac{1}{2} \left(\frac{t_f}{d} \right)^2}{(p' n) + \left(\frac{t_f}{d} \right)}$

$j d = d - \left(\frac{2 f_{m2} + f_{m1}}{3 (F'_m + f_{m1})} \right) t_f$

$k d \geq t_f$

$M_s = A_s f_s j d$

$M_m = \frac{1}{2} (f_{m1} + f_{m2}) A_f j d$

if over reinf. use $f_s = \left(\frac{1-k}{k} \right) \left(\frac{1}{3} F'_m \right) n$
if $f_s > 24000$ psi use $f_s = 24000$ psi

$F'_m = \frac{1}{3} F'_m$

over reinf $f'_m = \frac{1}{3} F'_m$
under reinf. $f'_m = \frac{24000(k)}{n(1-k)}$

$M_s = A_s F_s j d$

$F_s = 24000$

$M_m = \frac{1}{2} (f_{m1} + f_{m2}) A_f j d$

$F'_m = \frac{1}{3} F'_m$

$$\frac{M}{S} = \frac{My}{I}$$

$$F_s = 24000 \text{ psi for } f_y = 60 \text{ ksi}$$

$$F_b = \frac{1}{3} F'_m$$

$$E_m = 900 F'_m$$

$$E_s = 29000 \text{ psi}$$

$$n = 750 F'_m$$

Bearing walls

unreinforced

tension: $f_t = \frac{M}{S} - \frac{P}{A} \leq F_t = 0$ (dead load only)

compression: $\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0$ (1.33 for wind)

P+M From chart below

$$f_a = \frac{P}{A} = \text{psi}$$

$$f_b = \frac{M}{S} = \frac{My}{I} = \text{psi}$$

2.2.3.1

$$F_a = \frac{1}{4} F'_m \cdot R$$

$$\left(\frac{h}{r}\right) < 99 \quad R = 1 - \left(\frac{h}{r} \cdot \frac{1}{140}\right)^2$$

$$\left(\frac{h}{r}\right) > 99 \quad \text{if } P \leq \frac{1}{4} P_e = \frac{1}{4} \left[\frac{\pi^2 E_m I}{h^2} (1 - 0.577 \frac{e}{r})^3 \right]$$

$$\text{then } R = \left(\frac{70}{h/r}\right)^2 \quad \text{otherwise change walls}$$

reinforced CMU Brg Walls

2.3.3.2.2

Compressive stress in masonry $\leq 0.33 F'_m$

$$f_a < F_a$$

$$e_{min} = 0.1 t$$

$$M_{max} = (A_s F_s + P) j d = A'_s F_s j d$$

$$A'_s = A_s + \frac{P}{F_s}$$

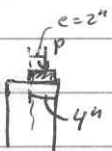
$$p'n = \frac{A'_s}{bd} n$$

$$k = \frac{p'n + \frac{1}{2} \left(\frac{t_f}{d}\right)^2}{j'n + \left(\frac{t_f}{d}\right)}$$

if $k d \leq t_f \Rightarrow j = 1 - \frac{k}{3}$

non-dim interaction diagram 12-24

$\frac{P}{F_b b t}$ } check tension controlled
 $\frac{P_e}{F_b b t^2}$ } + comp. controlled
 charts
 find P_{max}
 find $A_s = p b t$
 $g=0$ i.e. 1 layer of steel



load	mid-ht press		mid-ht suction	
	P	M = P(e)	P	M
DL self	$45 \frac{psf}{ft} \cdot \frac{h}{2} =$	0	same as press	0
DL net	(DL - uplift) trib _{roof}	$\frac{1}{2}(-P)(e)$	(DL - uplift) trib _{roof}	$\frac{1}{2}(-P)(e)$
DL tot	DL (trib _{roof})	$\frac{1}{2}(-P)(e)$	same as press	same as press
LL	LL (trib _{roof})	$\frac{1}{2}(-P)(e)$	"	"
WL	0	$\frac{(wL)(12 \frac{in}{ft})(h)^2}{8}$	"	$\frac{(wL)(12 \frac{in}{ft})(h)^2}{8}$

Load combo	Pressure	suction
DL self + DL tot	D+L	
DL self + DL net	D _n +L+W	
	D _n +W	

plot points

tek 14-19A

find $A_s = \frac{M_{max}}{f_t i^2}$ find moment capacities on non-brg charts $> M_{max}$ above (14-19A)

verify $P_{max} = \frac{1}{4} A_n F'_m R > P_{max}$ from above

pick $A_s = \frac{M_{max}}{f_t}$

check cases

$$A'_s = A_s + \frac{P}{F_s} \Rightarrow p'n = \frac{A'_s}{bd} n$$

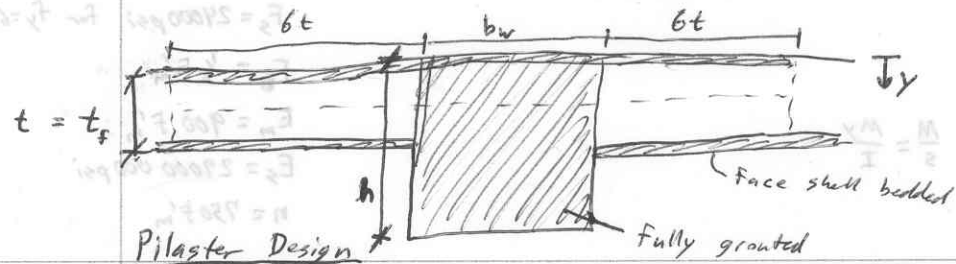
$$f_b = \frac{2 M_{max}}{\left(\frac{12 \frac{in}{ft}}{F_t}\right) j k d^2} \leq F_b = \frac{1}{3} f'_m$$

$$k = \frac{p'n + \frac{1}{2} \left(\frac{t_f}{d}\right)^2}{j'n + \left(\frac{t_f}{d}\right)} \rightarrow k d \leq t_f$$

$$f_s = \frac{M_{max}}{A'_s j d} \leq F_s = 24000 \text{ psi}$$

text interaction

12-24 $g=0$ (1 layer of steel) check P for tension controlled & comp controlled



$F_s = 29000 \text{ psi}$ for $f_y = 60 \text{ ksi}$
 $F_b = \frac{1}{3} f'_m$
 $E_m = 900 F'_m$
 $E_s = 29000000 \text{ psi}$
 $n = 21.5$ for $f'_m = 1500 \text{ psi}$
 $e_{min} = 0.1t$

Pilaster Design

effective flange projection = $6t$ (running bond)

find A_s & I

Part	A (in ²)	y (in)	Ay (in ³)	I_o (in ⁴)	$y - \bar{y}$ (in)	$A(y - \bar{y})^2$ (in ⁴)
Wall	$\frac{A_{block}}{ft} \cdot 12t$	$\frac{1}{2}t$	$A \cdot y$	$12t(I_o)_{face}$	$y - \bar{y}$ (see below)	$A(y - \bar{y})^2$
Pilaster	$b_w \cdot h$	$\frac{1}{2}h$	$A \cdot y$	$\frac{1}{12} b_w h^3$	$y - \bar{y}$	$A(y - \bar{y})^2$
sum	$\Sigma =$		$\Sigma =$	$\Sigma =$		$\Sigma =$

$\bar{y} = \frac{\Sigma Ay}{\Sigma A}$
 $I_{TOT} = I_o + \Sigma A(y - \bar{y})^2$

$r = \sqrt{\frac{I_{TOT}}{A_{TOT}}}$
 $\frac{h}{r} > 99 \quad P_n = \left(\frac{70}{(h/r)}\right)^2 \left(\frac{1}{4} F'_m A_n + 0.65 F_{sc} A_s\right)$ if unconf $A_s = 0$
 $\frac{h}{r} < 99 \quad P_n = \left(\frac{70}{(h/r)}\right)^2 \left(\frac{1}{4} F'_m A_n + 0.65 F_{sc} A_s\right)$

similar to brg walls

reinforced pilaster

① Tension in flange (suction)

$d = h - t_f - 1.5''$ cover + $\frac{1}{2}$ bar ϕ

$p_n = \frac{A_s}{b_w d} \cdot n$
 $k = -p_n + \sqrt{(p_n)^2 + 2p_n}$
 $j = 1 - \frac{k}{3}$

$M_s = A_s F_s j d$
 $M_m = \frac{1}{2} b j k d^2 F_b$
 $\left. \begin{matrix} M_s \\ M_m \end{matrix} \right\} \min = M_{max}$ if $M_s > M_m$ over reinf else under reinf

② Tension in web (pressure)

$d = h - t_f - 1.5''$

$p_n = \frac{A_s}{b_w d} n$
 $\frac{t_f}{d} =$

$k = \frac{p_n + \frac{1}{2} \left(\frac{t_f}{d}\right)^2}{p_n + \left(\frac{t_f}{d}\right)}$ $\Rightarrow k d \leq t_f$ if true, use eq's above

else ($k d > t_f$); $j d = d - \frac{(2 f_{m2} + f_{m1})}{3(f_{m2} + f_{m1})}$

$f_{m1} = \frac{1}{3} F'_m$

$f_{m2} = \left(\frac{k d - t_f}{k d}\right) F_{m1}$

$M_s = A_s F_s j d$

$M_m = \frac{1}{2} (f_{m1} + f_{m2}) (12t + b_w) (t_f) (j d)$
 $\left. \begin{matrix} M_s \\ M_m \end{matrix} \right\} \min = M_{max}$ if $M_s > M_m$ over reinf else under reinf

interaction diagrams: shortcut #1 (reinf. suction cases & web compression) if symmetric reinf. then web in comp is crit

- pilaster interaction reduces to rect. col. interaction

$A_s =$ $g = \frac{(h - 2(t_f))}{h} = 0.65 \Rightarrow$ use 0.6 interaction diagram solve for p_n that encompasses all of them

see brg wall on back

shortcut #2 straight line approx $\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0 \quad ; i_k + F_t \leq F_t ; i_k$

$e_{min} = 0.1t$

$A_g = b \cdot h$
 $A_n = A_g - A_{holes}$

bar	ϕ	Area
4	.5	.80
5	.625	.31
6	.75	.44
7	.875	.60
8	1.0	.79
9	1.128	1.0
10	1.27	1.27

$F_{sc} = 24000 \text{ psi}$ for $f_y = 60 \text{ ksi}$
 $F_b = \frac{1}{3} F'_m$
 $E_m = 900 F'_m$
 $E_s = 29000 \text{ 000 psi}$

Columns ($P_c = \frac{1}{4} F'_m A_n + 0.65 F_{sc} A_s$)

calculate $(\frac{h}{r})$

$r = \sqrt{\frac{I}{A}}$ (Tek 14-1A)

$h =$ unsupported height masonry steel

if $(\frac{h}{r} > 99)$ $P_c = (\frac{70}{(\frac{h}{r})^2}) (\frac{1}{4} F'_m A_n + 0.65 F_{sc} A_s) \leq \frac{1}{3} F'_m$

if $(\frac{h}{r} < 99)$ $P_c = (1 - (\frac{h}{r(140)})^2) (\frac{1}{4} F'_m A_n + 0.65 F_{sc} A_s) \leq \frac{1}{3} F'_m$

#4 \leq bar size \leq #11

Reinf. stl. (8" moduls)

$p_{min} = \frac{A_s}{A_g} = 0.0025 \leq p = \frac{A_s}{A_g} < 0.04$

$p_{max} = 0.0400$

ties (min size = $\frac{1}{4}$ ")

spacing $\begin{cases} 16 \text{ d bar} \\ 48 (\text{d tie}) \\ \text{min least col. dimension} \end{cases}$

column
 Max comp.
 load (axial)

min ecc = 0.1t

$P_c = R (\frac{1}{4} F'_m A_n + 0.65 F_{sc} A_s)$

design trial size $A_n = \frac{P_c}{\frac{1}{4} F'_m} = \dots \text{ in}^2 = (\dots \text{ in})^2 \leftarrow$ col dimensions for a square column (round to 8")

$A_{s, req} = \frac{(\frac{P_c}{R} - \frac{1}{4} F'_m A_n)}{0.65 F_{sc}} = \dots \text{ in}^2$ to get bar sizes $A_{size} \geq \frac{A_{s, req}}{\# \text{ cores}}$

$0.0025 < p = \frac{A_s}{A_g} < 0.04$

if $p > 0.04$ increase col. size
 if $p < 0.04$ increase steel size

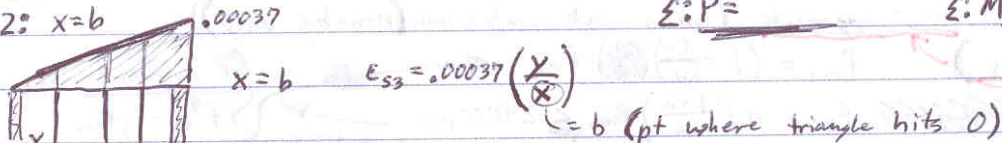
interaction diagram

case 1: $M=0$

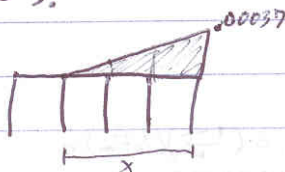
$P_o = F_b (A_n - A_{sc}) + F_c (A_{sc}) \cdot n$

tables for cases 2-5	element	Area involved	strain	Avg. stress	Force	ecc from \bar{c}	Moment
	Masonry	$x \cdot b$	$(\frac{1}{3} F'_m)_{typ} = 0.0037$ $E_m = 900 F'_m$ similar triangles	$\frac{1}{3} F'_m$ E_s stress (torc)	$A_m \cdot \text{stress}$	$\frac{2}{3} x - \frac{1}{2} \bar{c}$	$M_m = P_o \cdot e_m$
	A_{s1}	3bars (A_{bar})		E_s stress (torc)	$A_{s1} \cdot \text{stress}$	9"	$M_1 = P_o \cdot e_1$
	A_{s2}	2bars (A_{bar})		$\bar{c} = -$ stress	$A_{s2} \cdot \text{stress}$	0	$M_2 = P_o \cdot e_2$
	A_{s3}	3bars (A_{bar})		$\bar{c} = +$ stress	$A_{s3} \cdot \text{stress}$	-9"	$M_3 = P_o \cdot e_3$

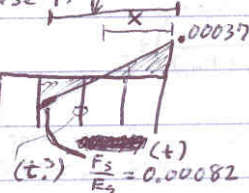
case 2: $x=b$



case 3:



case 4:



case 5:

$P=0$

$M_o = M_s = A_{s3} F_s j d$

$A_{s3} = 1$ layer only (3bars this layer)

$d_3 =$ dist. from comp face

Find $p_n = \frac{A_s}{bd}$

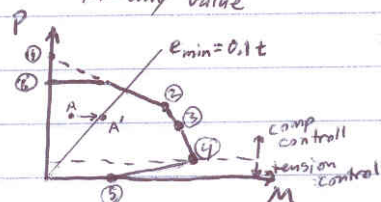
$K = -p_n + \sqrt{(p_n)^2 + 2p_n}$

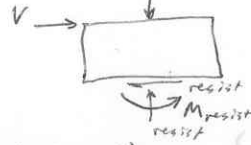
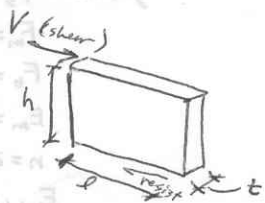
$j = 1 - \frac{K}{3}$

case 6: $P_{max} = P_c = R (\frac{1}{4} F'_m A_n + 0.65 F_{sc} A_s)$

$M =$ any value

text p. 12-12





gr. 60 $F_s = 24000 \text{ psi}$
 $F_m = \frac{1}{3} F'_m$
 $F_b = \frac{1}{3} F'_m$
 $E_m = 900 F'_m$
 $n = 21.5 \leftarrow \text{for } F'_m = 1500 \text{ psi}$
 $E_s = 29000000 \text{ psi}$
 $E_{\text{brick}} = 750 F'_m$

* watch control joint spacing
 ≠ 200' shear wall
 = (10) 20' shear walls

Shear walls (3 types, but start @ 1)

① Unreinforced

flexure: $f_t = \frac{M}{S_{\text{wall}}} - \frac{P}{A_{\text{wall}}} = \frac{6Vh}{tl^2} - \frac{P}{tl} \leq F_{t \text{ allow}} = 0$

shear: $F_v = 1.5 \frac{V}{bd} = 1.5 \left(\frac{V}{tl} \right)$

✓ $\frac{F_a}{F_a} + \frac{F_b}{F_b} \leq 1.0$ (see brg wall section & load combos)

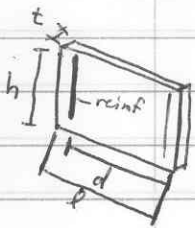
$\leq F_{v \text{ allow}} = \begin{cases} 120 \text{ psi} \\ 1.5 \sqrt{F'_m} \end{cases}$

$\min \left(v + 0.45 \left(\frac{P}{A_{\text{wall}}} \right) \right)$
 $v = 37 \text{ psi}$ (running bond w/ partial/no grout)

$\rightarrow v = 60 \text{ psi}$ (grouted)

$v = 15 \text{ psi}$ (ungrouted stack bond)

② Masonry reinf. in flexure, unreinf. in shear (ignore P)



flexure: $d = l \left(\frac{12''}{ft} \right) - n(8'')$ 8" block

$A_{s \text{ req}} = \frac{M}{F_s d(j)} \leq ()^{\#} \text{ bars} = A_s$
 assume 0.9

$pn = \frac{A_s}{td} (n)$

$k = -pn + \sqrt{(pn)^2 + 2(pn)}$

$j = 1 - \frac{k}{3}$

(check to see if A_s is ok)

$f_s = \frac{M}{A_s j d} \leq F_s$

$f_b = \frac{2M}{b k j d^2} \leq F_b$

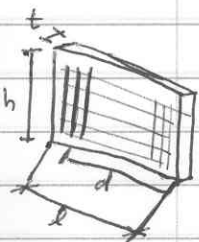
shear: $\frac{M}{Vd} =$

$f_v = \frac{V}{bd} = \frac{V}{td} \leq F_v$

for $\frac{M}{Vd} < 1.0$ $F_v = \frac{1}{3} \left(4 - \frac{M}{Vd} \right) \sqrt{F'_m} \leq (80 - 45 \left(\frac{M}{Vd} \right)) \text{ psi}$

for $\frac{M}{Vd} \geq 1.0$ $F_v = \sqrt{F'_m} \leq 35 \text{ psi}$

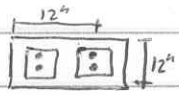
③ Masonry reinf. in Flexure and reinf. in shear (ignore P)



flexure: $d = l \left(\frac{12''}{ft} \right) - \text{Avg. reinf. layer}$ (8" increments for 8" blocks, 6" for 12" blocks)

check block size $f_v = \frac{V}{td} \leq F_v$ (see above)

if fails, $\begin{cases} \text{increase } F'_m \text{ or} \\ \text{increase block size (recommended)} \end{cases} \rightarrow 12'' \text{ blocks can do double bars}$
 new d value?



$A_{s \text{ req}} = \frac{M}{F_s d(j)} \leq ()^{\#} \text{ bars} = A_s$
 assume 0.9

$pn = \frac{A_s}{td} (n)$

$k = -pn + \sqrt{(pn)^2 + 2(pn)}$

$j = 1 - \frac{k}{3}$

(check to see if A_s is ok)

$f_s = \frac{M}{A_s j d} \leq F_s$

$f_b = \frac{2M}{b k j d^2} \leq F_b$

shear: (flexure d)

$A_{\text{horizontal}} = \frac{V(d)}{F_s \cdot d} =$ spacing $\leq \begin{cases} 24'' \\ \frac{1}{2} h_z \text{ (8'' increments)} \end{cases}$
 $\leq ()^{\#} \text{ bars} = A_s$ in bond beams
 2 max

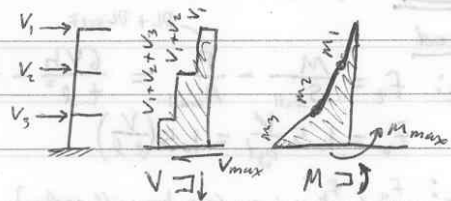
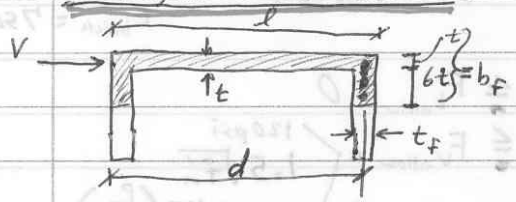
$f_v = \frac{V}{bd} = \frac{V}{td} \leq F_v$

for $\frac{M}{Vd} < 1.0$ $F_v = \frac{1}{2} \left(4 - \frac{M}{Vd} \right) \sqrt{F'_m} \leq (120 - 45 \left(\frac{M}{Vd} \right)) \text{ psi}$

for $\frac{M}{Vd} \geq 1.0$ $F_v = 1.5 \sqrt{F'_m} \leq 75 \text{ psi}$

gr. 60 $F_s = 24000 \text{ psi}$
 $F_m = 1/3 F'_m$
 $F_b = 1/3 F'_m$
 $E_m = 900 F'_m$
 $n = 21.5 \rightarrow \text{for } F'_m = 1500$
 $F_{brick} = 750 F'_m$
 $F_s = 29000,000 \text{ psi}$

Flanged Shear wall (non-bearing)



design for max V & M_{max}

$$d = l(12\frac{t_w}{t_f}) - \frac{1}{2}(t)$$

$$A_s \text{ req} = \frac{M}{F_s d(j)} \leq (\quad)^{\#} \text{ bars} = A_s$$

assume 0.9

$$pn = \frac{A_s}{b_f d}$$

$$k = \frac{pn + \frac{1}{2}(\frac{t_f}{d})^2}{(pn + (\frac{t_f}{d}))}$$

$$kd = \quad \geq t_f \text{ ok, it is a T-beam}$$

$$jd = d - \left(\frac{2 f_{m2} + f_{m1}}{3(f_{m2} + f_{m1})} \right) t_f$$

$$F_s = n \left(\frac{1-k}{k} \right) (1/3 F'_m) < 24000 \text{ psi} \rightarrow \text{if true: } f_{m1} = 1/3 F'_m$$

if false: $f_{m1} = \frac{24000(k)}{n(1-k)}$ (under reinf)

$$f_{m1} \rightarrow$$

$$f_{m2} = \left(1 - \frac{t_f}{kd} \right) f_{m1}$$

$$M_s = A_s F_s (jd) \geq M_{max}$$

$$M_m = \frac{1}{2} (f_{m1} + f_{m2}) (b_f \cdot t_f) (jd) \geq M_{max}$$

shear check: $\frac{M}{Vd} = ?$

$$F_v = \frac{V}{t_f d} \leq F_v$$

$$\text{for } \frac{M}{Vd} \leq 1 \quad F_v = \frac{1}{3} \left(4 - \frac{M}{Vd} \right) \sqrt{F'_m} \leq (80 - 45 \frac{M}{Vd}) \text{ psi}$$

$$\text{for } \frac{M}{Vd} \geq 1 \quad F_v = \sqrt{F'_m} \quad (39) \leq 35 \text{ psi}$$

Bar size	diameter	Area
3	0.375"	0.11 in ²
4	0.5"	0.20 in ²
5	0.625"	0.31 in ²
6	0.750"	0.44 in ²
7	0.875"	0.60 in ²
8	1.00"	0.79 in ²
9	1.128"	1.0 in ²
10	1.27"	1.27 in ²
11	1.41"	1.56 in ²
14	1.693"	2.25 in ²
18	2.257"	4.0 in ²

Earthquake

$$g = 32.2 \text{ ft/s}^2$$

$$g = 386.4 \text{ in/s}^2$$

EQ difference

- ① Peak Amplitude
- ② duration of strong motion
- ③ frequency content = $\frac{1}{T}$

⑩ waves

↔ P : compressional 19000 ft/s
 ~~~ S : shear waves 10000 "  
 ↔ R/L : 9000 "

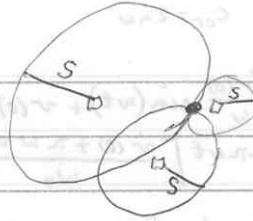
## ⑪ locating the Epicenter $S = t_s - t_p$

## ⑫ Magnitude

$$M = \log_{10} \left( \frac{A}{A_0} \right)$$

$$A_0 = 0.001 \text{ mm}$$

A = peak amplitude 100 km from Epicenter



## ⑬ energy release

$$E = 10^{(11.8 + 1.5M)} \text{ in erg} = 10^{-7} \text{ Joules}$$

## ⑭ length of active Faults

$$L = 10^{(1.02M - 5.77)}$$

## ⑮ (PGA) peak ground acceleration

$$PGA = 15 \frac{\text{ft/s}^2}{2} = \frac{15 \text{ ft/s}^2}{32.2 \text{ ft/s}^2} = 0.47g \quad (47\%g)$$

## ⑯ intensity

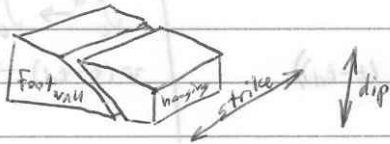
(MMI) modified mercalli intensity = I → XII

| MMI  | PGA           |
|------|---------------|
| VI   | 0.08g → 0.15g |
| VIII | 0.24g → 0.45g |
| X    | 0.6g → 0.8g   |

## ⑰ vertical peak ground acc. (PGA<sub>vert</sub>)

$$PGA_{\text{vert}} = \frac{1}{3} PGA$$

## Faults



Fault wall controls direction



## ⑳ frequency of occurrence (#/yr)

$$N(\#/yr) = 10^{(a-bM)} = \frac{\# \text{ of EQ of } M \text{ magnitude}}{\text{yr}}$$

## ㉑ Probability of occurrence $\frac{\# \text{ Magnitude}}{x \text{ (years)}}$

$$N = cye^{-\frac{m}{B}}$$

$$P = K \Delta$$

$$\omega = \sqrt{\frac{K}{m}} = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \quad \tau = \frac{P_0 \sin \theta}{2Bpk}$$

$$\tau = \frac{c}{C_{cr}} = \frac{\delta}{2\pi} \quad C_{cr} = 2m\omega$$

springs parallel:  $K_{eq} = k_1 + k_2$

springs series:  $\frac{1}{k_T} = \frac{1}{k_1} + \frac{1}{k_2}$

Free vibration

$$\tau = \frac{c}{C_{cr}} = \frac{\delta}{2\pi} = \frac{1}{2\pi} \left( \frac{v_1}{v_2} \right)$$

$$C_{cr} = 2m\omega$$

| Column CASE       | $\Delta$                      | K                            | beam cage | $\Delta$              | K                    |
|-------------------|-------------------------------|------------------------------|-----------|-----------------------|----------------------|
|                   | $\frac{Ph}{AE}$               | $\frac{AE}{h}$               |           | $\frac{Pl^3}{48EI}$   | $\frac{48EI}{l^3}$   |
|                   | $\frac{Ph^3}{3EI}$            | $\frac{3EI}{h^3}$            |           | $\frac{5wl^4}{384EI}$ | $\frac{384EI}{5l^3}$ |
|                   | $\frac{Ph^3}{12EI}$           | $\frac{12EI}{h^3}$           |           | $\frac{Pl^3}{192EI}$  | $\frac{192EI}{l^3}$  |
|                   | $\frac{wh^4}{8EI}$            | $\frac{8EI}{h^3}$            |           | $\frac{wl^4}{384EI}$  | $\frac{384EI}{l^3}$  |
| <b>Brace Case</b> | $\Delta$                      | K                            |           |                       |                      |
|                   | $\frac{Pl}{AE \cos^2 \theta}$ | $\frac{AE \cos^2 \theta}{l}$ |           |                       |                      |

smaller mass  $\rightarrow$  higher frequency  
 larger stiffness  $\rightarrow$  higher frequency

eq. of motion  $m\ddot{v}(t) + c\dot{v}(t) + kv(t) = P(t)$

$\begin{cases} P(t) = 0 & \text{Free vibration} \\ P(t) = P_0 \sin(\omega t) & \text{harmonic} \end{cases}$

$c_{cr} = 2m\omega$

SDOF

Un damped  $v(t) = \frac{\dot{v}(0)}{\omega} \sin(\omega t) + v(0) \cos(\omega t)$

damped (circular)  $v(t) = e^{-\pi \omega t} \left[ \frac{\dot{v}(0) + \pi \omega v(0)}{\omega_0} \sin(\omega_0 t) + v(0) \cos(\omega_0 t) \right]$

damped circular frequency  $\omega_0 = \omega \sqrt{1 - \pi^2}$  see notes

Harmonic - SDOF

undamped  $v(t) = \underbrace{\left[ \frac{\dot{v}(0)}{\omega_n} - \frac{P_0}{k} \frac{\beta}{1 - \beta^2} \right] \sin(\omega_n t) + v(0) \cos(\omega_n t)}_{v_c(t) \text{ (Free vibration transient)}} + \underbrace{\frac{P_0}{k} \frac{1}{1 - \beta^2} \sin(\omega t)}_{v_p(t) \text{ (steady state)}}$

$B = \frac{\omega}{\omega_n}$  frequency ratio

$v_{stat} = \frac{P_0}{k}$  static displacement

$R(t) = \frac{v(t)}{v_{stat}} = \frac{1}{(1 - \beta^2)} (\sin(\omega t) - \beta \sin(\omega_n t))$  response ratio

damped  $v(t) = e^{-\pi \omega_n t} \left[ \frac{\dot{v}(0) + v(0)\pi}{\omega_n} \sin(\omega_n t) + v(0) \cos(\omega_n t) \right] + \frac{P_0}{k} \frac{1}{\sqrt{(1 - \beta^2)^2 + (2\pi\beta)^2}} \left[ (1 - \beta^2) \sin(\omega t) - 2\pi\beta \cos(\omega t) \right]$

$v_p(t) = p \sin(\omega t - \theta)$

@ resonance  $\beta = 1$

$\pi = \frac{P_0 \sin \theta}{2B\beta k}$

$p = \frac{P_0}{k} \frac{1}{\sqrt{(1 - \beta^2)^2 + (2\pi\beta)^2}}$

$DA = \left( \frac{p}{P_0/k} \right) = \frac{1}{\sqrt{(1 - \beta^2)^2 + (2\pi\beta)^2}}$

$DA = \frac{1}{2\pi}$

Rect. Impulse ( $t_d < T_n$ ) ( $v(0) = 0$ )

undamped  $v(t - t_d) = \frac{1}{m\omega} \left[ \int_0^{t_d} p(t) dt \right] \sin(\omega(t - t_d))$  w/  $v(0) = 0$   
 assumes  $v(0) = 0$

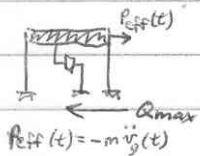
damped  $v(t - t_d) = \frac{1}{m\omega_0} \left[ \int_0^{t_d} p(t) dt \right] e^{-\pi \omega_0(t - t_d)} \sin(\omega_0(t - t_d))$   
 $\omega_0 = \omega \sqrt{1 - \pi^2}$

total response

$t_d \rightarrow \gamma$   
 $\int_0^{t_d} \rightarrow \int_0^t$

$v(t - t_d) \rightarrow v(t)$

Ground displacement motion



$D = S_d = \text{spectral displacement} = v_{max}(t)$

$S_d = Q_{max} \left( \frac{1}{m\omega_n^2} \right)$

$V = S_{pv} = \text{spectral pseudo-velocity} = V_{max}(t)$

$S_{pv} = \omega_n S_d$

$A = S_{pa} = \text{spectral pseudo-acceleration} = \ddot{v}_{tot max}(t)$

$S_{pa} = \omega_n^2 S_d$  (units are  $X \cdot g$ )

$Q_{max} = W \left( \frac{A}{g} \right)$

note: A is in units of g, so the g's cancel out

watch charts (ie. if chart is  $\ddot{v} = 1g$  & have  $\ddot{v} = .5g$  values are  $A_i = \frac{1}{2} A_{table}$ )

| Diaphragm | Distribution |
|-----------|--------------|
| rigid     | stiffness    |
| flexible  | trib. area   |



Load combinations

1.0L if LL > 100psf

5.  $1.2D + 1.0E + 0.2S + 0.5L$

$E = p Q_E + 0.2 S_{Ds} \cdot D$  (only w/ load combo 5)

7.  $0.9D + 1.0E + 1.6H$

$E = p Q_E - 0.2 S_{Ds} \cdot D$  (only w/ load combo 7)

ground pressure

redundancy ( $1.0 \leq p_x \leq 1.5$ )

for SDC = A, B, or C

$p = 1.0$

for SDC = D, E, or F

$p = p_x \cdot \max$  of all floor levels

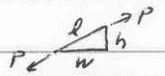
$1.0 \leq p_x = 2 - \frac{20}{r_{max} \cdot \sqrt{A_x}} \leq 1.5$

$A_x$  = floor area in  $ft^2$

$Q_E$  = member force

reliability Factor

$r_{max} = \frac{(V_{element})_{max}}{V_{story}}$



Braced frame:  $r_{max} = \frac{(V_{brace})_{max}}{V_{story}}$

$V_{brace} = (\text{axial load}) \cdot (\frac{w}{L})$

\* Moment frame:  $r_{max} = \frac{(V_{col1} + V_{col2})_{max}}{V_{story}}$

adjacent cols use 0.7  $V_{col}$  if columns are common to 2 bays.

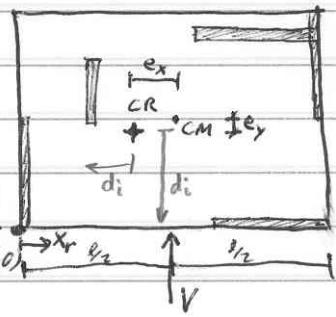
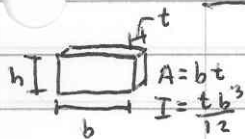
Shear wall:  $r_{max} = \frac{(V_{wall} \cdot (\frac{l_w}{L_{wall}}))_{max}}{V_{story}}$

$l_{wall}$  = wall length (ft)  $\frac{l_w}{L} \leq 1.0$

rigidity  $\Delta = \frac{Ph^3}{12EC} + 1.2 \frac{Ph}{AG}$   $G = 0.4E$

$R_{fix} = \frac{k}{E} = \frac{1}{i(\frac{h}{b})^3 + 3(\frac{h}{b})}$

Shear in building due to torsion & direct load (EQ loading)



Center of mass - @ center of building  $w/4 = R_{center}$

center of rigidity:  $X_r = \frac{\sum R_{ix} X_i}{\sum R_{ix}}$

$Y_r = \frac{\sum R_{iy} Y_i}{\sum R_{iy}}$

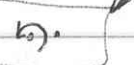
$e_{min} = \pm 0.05L$

$e_x = X_{cm} - X_{cr}$

$e_y = Y_{cm} - Y_{cr}$



$T = V(e)$



torsional moment of inertia

$J = \sum (R_i (d_i)^2)$   $T = V(e \pm e_{min})$

Direct shear:  $V_i = \frac{R_i}{\sum R_i} V_{story}$

Torsional shear:  $V_{T,i} = \frac{T_i \cdot d_i \cdot R_i}{J}$  (check if you add or subtract  $e_{min}$ )

$V(e \pm e_{min})$

$T_{resist} \uparrow \downarrow V_{resist} \Rightarrow$  subtract  $e_{min}$

$T_{resist} \downarrow \downarrow V_{resist} \Rightarrow$  add  $e_{min}$

Total  $V_i = V_{i,direct} \pm V_{i,torsional}$

tabk 9.5.2.3.2

check plan irregularity



$V = RS = S = \frac{V}{R}$

item 1a: if  $\Delta_{max} \geq 1.2 (\frac{\Delta_1 + \Delta_2}{2})$  then irregular, if  $> 1.4 (\frac{\Delta_1 + \Delta_2}{2})$  then extreme irregularity

9.5.5.5.2

if irregularity: Amplification Factor (SDC C, D, E, F)

$1.0 \leq A_x = \left( \frac{S_{max}}{1.2(S_{avg})} \right)^2 \leq 3.0$  (ignore if light framed)

$e_{min2} = A_x \cdot e_{min}$

$V_{T,i} = \frac{V(e \pm e_{min2}) d_i R_i}{J} \Rightarrow$  new higher  $V_{Tors,i} = V_{dir,i} + V_{T,i}$

Component Design

$$F_p = \frac{0.4 a_p S_{DS} W_p}{(R_p / I_p)} \left( 1 + 2 \left( \frac{z}{h} \right) \right) \quad 0.3 S_{DS} I_p W_p \leq F_p \leq 1.6 S_{DS} I_p W_p$$

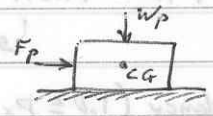
table 9.6.2.2

$a_p$  = component amplification factor  $1.0 \leq a_p \leq 2.5$

$R_p$  = component response modification factor

$z$  = avg height in structure of attachment point of component

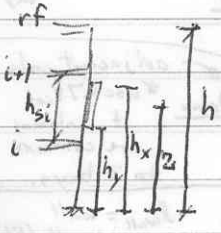
$h$  = avg roof height  $W_p$  = weight of component  $I_p$  = importance factor



relative displacement requirement

$$D_p = \delta_{xA} - \delta_{yA} \leq (h_x - h_y) \frac{\Delta_{allow}}{h_{si}}$$

( $\Delta$  @ level x determined by elastic analysis & multiplied by  $C_d$ )



Overturning moment

$$M_o = \sum (F_x (h_x - h_o))$$

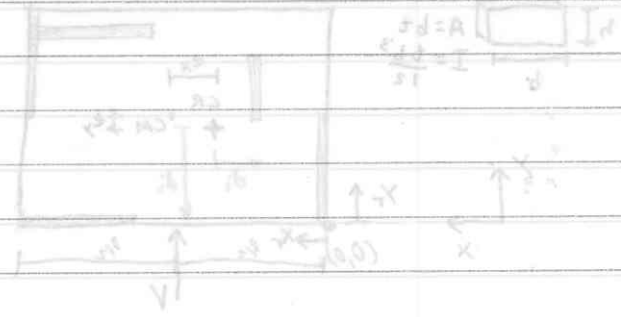
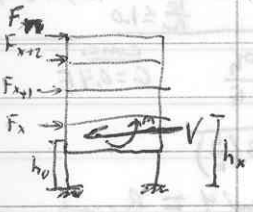
$$V_o = \sum F_x$$

story drift

table 9.5.2.8

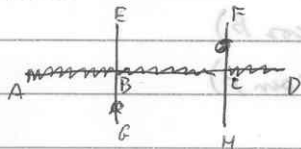
$$\Delta_x = \delta_{x+1} - \delta_x \leq \Delta_{allow} \approx \frac{h}{400}$$

$$\delta_x = \frac{C_d \delta_{xe}}{I} \quad \text{elastic analysis}$$



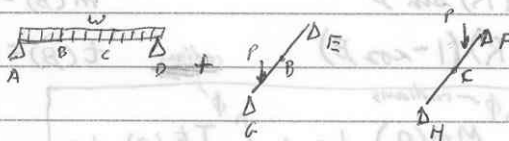
*(Faint handwritten notes and diagrams, including equations like V\_i = T\_i \cdot h\_i / R\_i, V = V\_i, and various structural analysis diagrams.)*

## Grid structure (consist. def.)



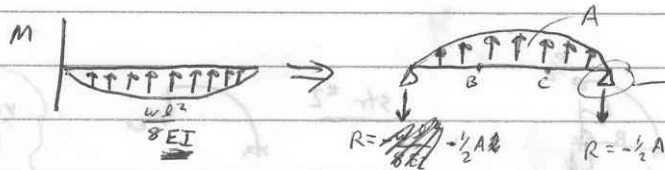
$$\Delta c_1 + \Delta e_2 = 0$$

$$\Delta b_1 + \Delta b_2 = 0$$



use conjugate beam

| Real       | Conj. |
|------------|-------|
| pin roller | Pin   |
| Fixed      | Free  |
| Free       | Fixed |

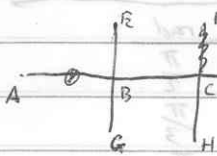


watch and conditions

$$\Delta_B = \frac{M_B}{EI} = \Delta_B \quad (\text{find internal moment @ pt. } = \Delta_B)$$

## Grids (by slope deflection)

use modified eq. if possible



(no torsion)

$$\begin{cases} M_{BA} = \\ M_{BC} = \\ M_{CB} = \\ M_{CD} = \end{cases}$$

$$\begin{cases} M_{BE} \\ M_{BF} \\ M_{CF} \\ M_{CH} \end{cases}$$

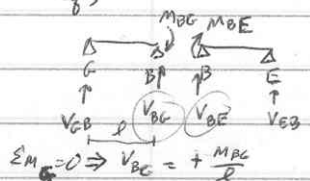
$$\begin{cases} V_{BA} \\ V_{BC} \\ V_{BE} \\ V_{BF} \end{cases} \quad \begin{cases} V_{CB} \\ V_{CD} \\ V_{CG} \\ V_{CH} \end{cases}$$

use L for each member

equilibrium eq. (Known)

$$\begin{cases} \sum M_{Bx} = 0 = M_{BE} + M_{BF} \\ \sum M_{By} = 0 = M_{BA} + M_{BC} \\ \sum M_{Cx} = 0 = M_{CB} + M_{CD} \\ \sum M_{Cy} = 0 = M_{CH} + M_{CF} \\ \sum F_{Bz} = 0 = V_{BA} + V_{BC} + V_{BE} + V_{BF} \\ \sum F_{Cz} = 0 = V_{CB} + V_{CH} + V_{CD} + V_{CF} \end{cases}$$

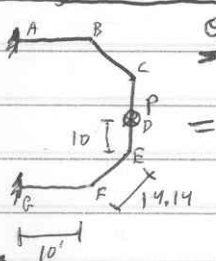
Joint reaction eq's



## Awning beams (use symmetry)

$$\Theta_1 + \Theta_2 = 0$$

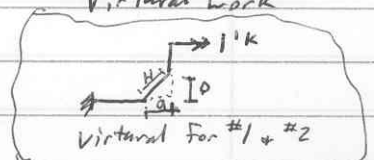
$$\Theta_i = \int W_i = \int_0^x \frac{M_m}{EI} dx + \int_0^x \frac{T \pm}{GJ} dx$$



Structure #1

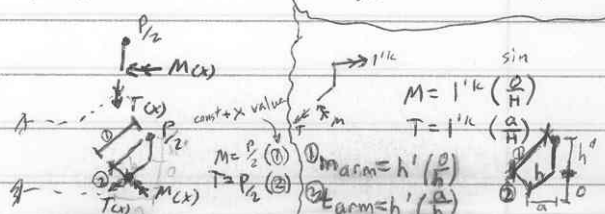
Structure #2

since finding  $\Theta$  Virtual work



| str #1  | str #2       | real structure M | virtual structure m |
|---------|--------------|------------------|---------------------|
| memb DE | X 0 → 10'    |                  |                     |
| memb EF | X 0 → 14.14' |                  |                     |
| memb FG | X 0 → 10'    |                  |                     |

\*  $M @ pt E = M_{str 1} + M_{str 2}$  ← real components



Curved beam in plan (const radius) (Radians)

$M(\beta) = -P(R) \sin \beta$

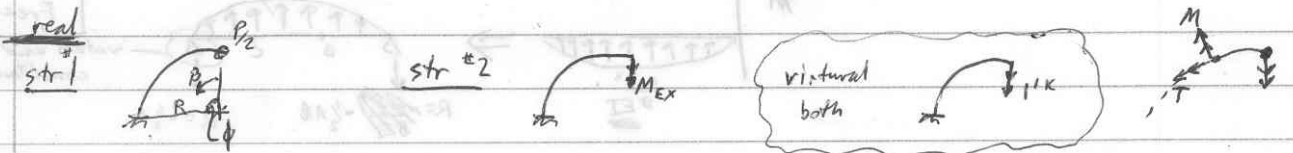
$T(\beta) = P(R)(1 - \cos \beta)$

$m(\beta) = P(R) \cos \beta$

$t(\beta) = P(R) \sin \beta$

pt load  
if  $M_{EX}$  just multiply virtual by  $M_{EX}$

$$\theta_c = w_i = \int_0^\phi \frac{Mm(R)}{EI} d\beta + \int_0^\phi \frac{Tt(R)}{EJ} d\beta$$



Calculus

$\int_0^\phi \sin \beta d\beta = -\cos \phi + 1$

$\int_0^\phi \sin^2 \beta d\beta = \frac{1}{2}(\phi - \sin \phi \cos \phi)$

$\int_0^\phi \cos \beta d\beta = \sin \phi$

$\int_0^\phi \cos^2 \beta d\beta = \frac{1}{2}(\phi + \sin \phi \cos \phi)$

$\int_0^\phi \sin \beta \cos \beta d\beta = \frac{\sin^2 \phi}{2}$

| deg | rad             |
|-----|-----------------|
| 90  | $\frac{\pi}{2}$ |
| 60  | $\frac{\pi}{3}$ |
| 45  | $\frac{\pi}{4}$ |
| 30  | $\frac{\pi}{6}$ |

| member | $\beta$ | real                          |   | virtual |   |
|--------|---------|-------------------------------|---|---------|---|
|        |         | M                             | T | m       | t |
| str #1 | EA      | $0 \rightarrow \frac{\pi}{2}$ |   |         |   |

$M_B = M_{str \#1} + M_{str \#2}$  (M @ a point = sum of the real components at that pts)  
 $T_B = T_{str \#1} + T_{str \#2}$

$\beta = \phi$  @ pt B in radians



| Real   | Conjugate |
|--------|-----------|
| Pin    | Pin       |
| Fixed  | Free      |
| Free   | Fixed     |
| Roller | Pin       |

### Virtual Work



axial:

$$1^k \Delta = \sum \frac{f_i F_i l_i}{A_i E_i}$$



flexural:

$$1^k \Delta = \int_0^l \frac{m_i M_i}{E_i I_i} dx$$

Deep beams:  $1^k \Delta = \int_0^l \frac{k_i v_i V_i}{A_i G_i} dx$

$$G_i = \frac{E}{2(1-\nu)}$$

$\nu = 0.15$  for conc  
 $\nu = 0.29$  for steel

$k_i =$  shape factor

$k_i = 1.2$  for rectangles  
 $k_i = 1.0$  for W-sections w/A = Au

Torsional:

$$1^k \Delta = \int_0^l \frac{t_i T_i}{k_i G_i} dx$$

note: torsion is constant throughout member; E or G can use super position

### Slope Deflection

see handout

find known conditions @ joint 3  $\sum M = 0 = M_{34} + M_{31}$

### Lateral Force resist

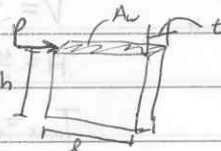
(add  $\Delta$  not k's)

shear walls

$$A = \frac{Ph^3}{3EI} + \frac{2.78 Ph}{t l E}$$

$F = k \Delta$

flexure ( $\frac{h}{l} > 3$ )  
shear ( $\frac{h}{l} < 0.3$ )



$\frac{h}{l} < 0.3$  short wall

$\frac{h}{l} > 3$  tall wall

$$I = \frac{1}{12} t l^3$$

tall walls ( $\frac{h}{l} > 3$ )

$$k = \frac{3E}{h^3} \left( \frac{t l^3}{12} \right)$$

short walls  $\frac{h}{l} < 0.3$

$$k = \frac{t l E}{2.78 h}$$

if const t & E

$$k = \frac{l^3}{4 h^3}$$

$$k = \frac{l}{2.78 h}$$

if all tall/short

$$k = l^3$$

$$k = l$$

$0.3 < \frac{h}{l} < 3$  (Both)

$$k = \frac{1}{\frac{Ph^3}{3EI} + \frac{2.78 h}{t l E}}$$

if ( $\frac{t + E h}{const}$ )

$$k = \frac{1}{\frac{4 h^2}{l^3} + \frac{2.78}{l}}$$

### Stiffness analysis

$$K_{xx}^y = K_{yy}^x = -K_{xy} = -K_{yx}$$

vertical members  $K_{xx}^y = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \frac{AE}{L}$

horizontal members  $K_{xx}^y = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \frac{AE}{L}$

if  $x \begin{matrix} / \\ s \\ \backslash \\ l \end{matrix} y \begin{matrix} / \\ h \\ \backslash \end{matrix}$   $K_{xx}^y = \begin{bmatrix} (\frac{l}{s})^2 & (\frac{2h}{s^2}) \\ (\frac{2h}{s^2}) & (\frac{h^2}{s^2}) \end{bmatrix} \frac{AE}{L}$

if  $x \begin{matrix} / \\ h \\ \backslash \\ l \end{matrix} y \begin{matrix} / \\ s \\ \backslash \end{matrix}$   $K_{xx}^y = \begin{bmatrix} (\frac{l}{s})^2 & -(\frac{2h}{s^2}) \\ -(\frac{2h}{s^2}) & (\frac{h^2}{s^2}) \end{bmatrix} \frac{AE}{L}$

$$0 = \begin{Bmatrix} P_{ix} \\ P_{iy} \\ P_{ix} \\ P_{iy} \end{Bmatrix} = AE \begin{bmatrix} \sum k_{11} & k_{12} \\ k_{21} & \sum k_{22} \end{bmatrix} \begin{Bmatrix} \Delta_{ix} \\ \Delta_{iy} \\ \Delta_{ix} \\ \Delta_{iy} \end{Bmatrix} = 0$$

member force

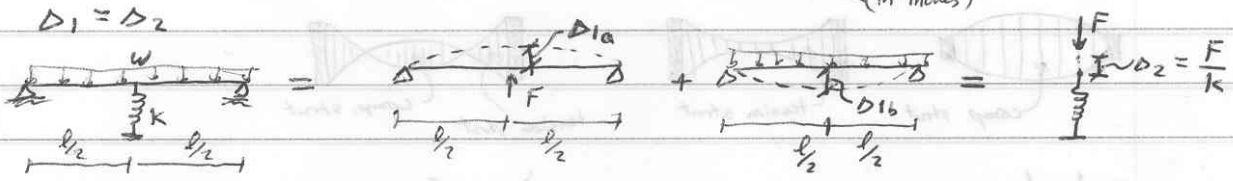
$$\begin{Bmatrix} f_{xy} \\ f_{yx} \end{Bmatrix} = \begin{bmatrix} k_{xx} & -k_{xy} \\ -k_{xy} & k_{yy} \end{bmatrix} \begin{Bmatrix} \Gamma_{xy} & 0 \\ 0 & \Gamma_{xy} \end{Bmatrix} \begin{Bmatrix} \Delta_{ix} \\ \Delta_{iy} \\ \Delta_{ix} \\ \Delta_{iy} \end{Bmatrix}$$

$$k = \frac{AE}{L} \quad \Gamma_{xy} = \begin{bmatrix} l & h \\ s & s \end{bmatrix}$$

# Elastic Supports (Consistent Deformation)

$$K_{\text{cable}} = \frac{AE}{l} \quad \left(\frac{K}{\text{in}}\right)$$

$$K_{\text{col}} = \frac{AE}{l} \quad \left(\frac{K}{\text{in}}\right)$$



$$D_1 = D_2$$

$$+ \downarrow D_1 = D_{1b} - D_{1a} = D_2 = \frac{F}{K}$$

$$\frac{5wl^4}{384EI} - \frac{Fl^3}{48EI} = \frac{F}{K}$$

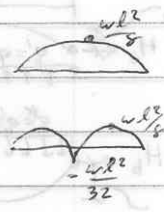
steel manual equations @ each location

$$M_{@l/2} = \frac{wl^2}{8} - \frac{Fl}{4}$$

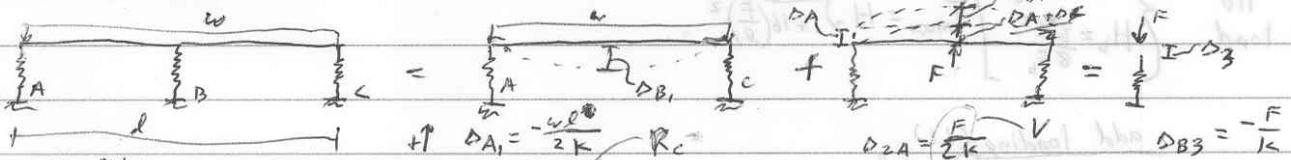
dist load w/l values shown above

let  $r = \frac{K}{(EI/l^3)}$

$$F = \frac{wl}{2} \left( \frac{5}{4} \cdot \frac{r}{(r+48)} \right)$$

$$M_{@l/2} = \frac{wl^2}{8} \left( 1 - \frac{5}{4} \cdot \frac{r}{(r+48)} \right)$$


## 3 elastic supports



if  $r = \frac{K}{(EI/l^3)}$

| r     | exterior support | int support | middle support |
|-------|------------------|-------------|----------------|
| 1     | .195P            | .202P       | .206P          |
| 10    | .189P            | .218P       | .250P          |
| 100   | .115P            | .270P       | .430P          |
| 1000  | -.488P           | .210P       | .676P          |
| 10000 | -.913P           | .051P       | .925P          |

$$D_{C1} = \left( \frac{wl^4}{2} \right) / K$$

$$D_{B1} = -\frac{5wl^4}{384EI} - \left( \frac{wl}{2K} \right) \cdot 2$$

$$D_{A2} = \frac{F}{2K}$$

$$D_{C2} = \left( \frac{F}{2} \right) / K$$

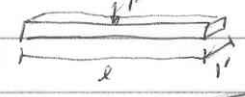
$$D_{B2} = \frac{Fl^3}{48EI} + \left( \frac{F}{2K} \right) \cdot 2$$

avg reaction force displacement

rigid beam, Flex. support (good) Flex. support beam (bad)

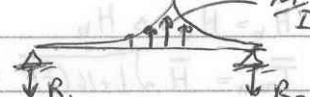
## Elastic Foundation iterations ( $\sigma = D \cdot E_{\text{soil}}$ )

start w/ uniform soil pressure

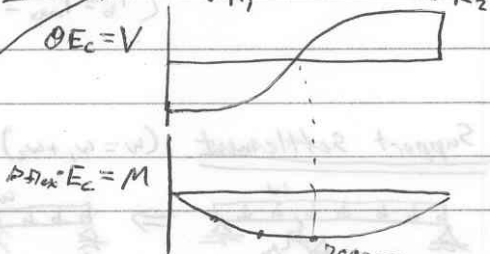
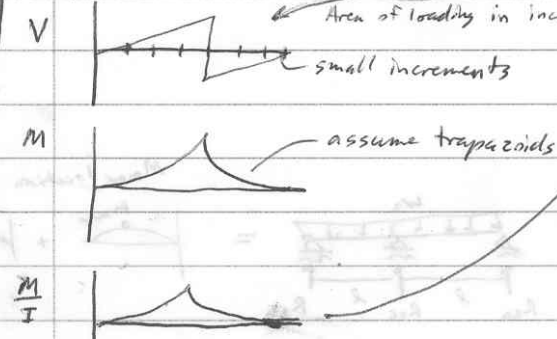


$$w = \frac{P}{l} = p \cdot b$$

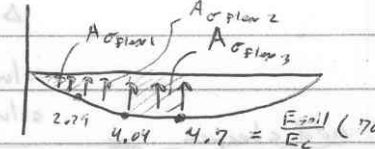
conjugate beam



break into smaller sections (ie 1', 2')



$$\sigma_{\text{flex}} = \frac{E_{\text{soil}}}{E_c} (D_{\text{flex}} \cdot E_c)$$



$$\sum \sigma_{\text{flex}} = 2 \sum (A_{\text{flex}1} + A_{\text{flex}2} + A_{\text{flex}3})$$

up to E (2) = double if symmetrical

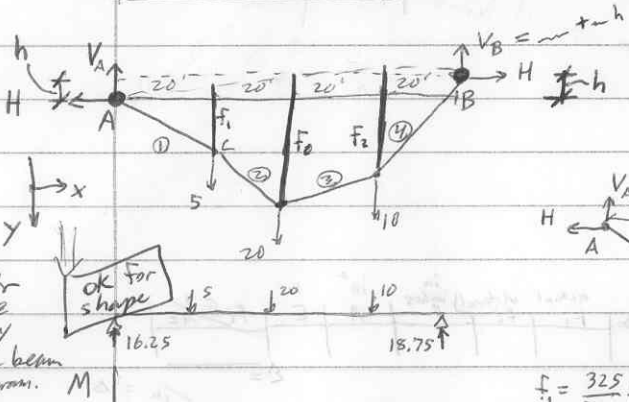
Iteration #2 (Avg of start + finish @ each point)



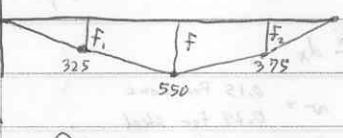
$$\sigma_{\text{uniform}} = \frac{P - \sum \sigma_{\text{flex}}}{l} = 2.11 \text{ KIF}$$

effective range ~~0.03 < n < 0.15~~  
 $0.03 < n < 0.15$

### Tension Structures (Point loads) ( $n = \frac{F}{H}$ )



cable str shape same idea w/ simple beam moment diagram. M  
 deflected shape is M diagram from simple beam



③  $T_{max} = \sqrt{(V_B)^2 + (H)^2}$   
 max reaction (shear)

②  $\sum M_c = 0$   
 $H = \frac{V_A \cdot X_1}{F_1}$

① solve for f

$f_1 = \frac{325}{550} F \leq F$   
 $f_2 = \frac{375}{550} F \leq F$

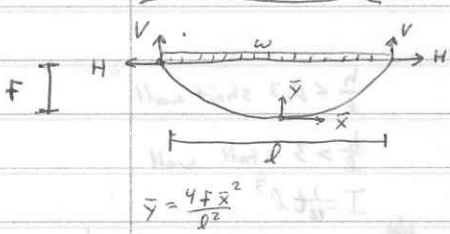
$L_{TOT} = l_1 + l_2 + l_3 + l_4 =$   
 $\sum = \sqrt{(20')^2 + (\frac{325}{550} F)^2} + \sqrt{(20')^2 + (F - \frac{325}{550} F)^2} + \dots$   
 given  $(x)^2 + (f_1)^2$        $(x)^2 + (f - f_1)^2$

### ④ second order effects

| member | x   | y         | $l_n = \sqrt{x^2 + y^2}$ | $T = H(\frac{A_n}{x})$ | elongation<br>$\Delta = \frac{T l_n}{AE}$ |
|--------|-----|-----------|--------------------------|------------------------|-------------------------------------------|
| ①      | 20' | $f_1$     |                          |                        |                                           |
| ②      | 20' | $f - f_1$ |                          |                        |                                           |
| ③      | 20' | $f - f_2$ |                          |                        |                                           |
| ④      | 20' | $f_2$     |                          |                        |                                           |

$\sum \Delta \approx L_{TOT} \cdot \epsilon$   
 $\epsilon = \Delta_{TOT} \Rightarrow l_{initial} = L_{TOT} - \Delta_{TOT}$

### Tension Structures (distributed loads)



$V = \frac{w \cdot l}{2}$   
 $H = \frac{w l^2}{8f}$   
 $T_{max} = \sqrt{(V)^2 + (H)^2} =$   
 $T_{max} = \frac{w l}{8n} \sqrt{16n^2 + 1}$

iterative process to find F ( $L_{initial} = l [1 + \frac{8}{3} n^2 - \frac{32}{5} n^4 + \dots]$ )  $n_i = \frac{F_i}{H}$

$F_0 = F_{initial}$  (unloaded cable sag) approximation  $L_{final} \Rightarrow n_f = \frac{F}{H}$

$n = \frac{F}{H}$   
 $H = \frac{w l^2}{8 F_n}$   
 $\Delta L = \frac{H l}{AE} [1 + \frac{16}{3} n^2]$   
 $\Delta F = \frac{15 (\Delta L)}{16 n (5 - 24 n^2)}$

$F_{TOT} = F_{initial} + \Delta F = F_0 + \Delta F_n$

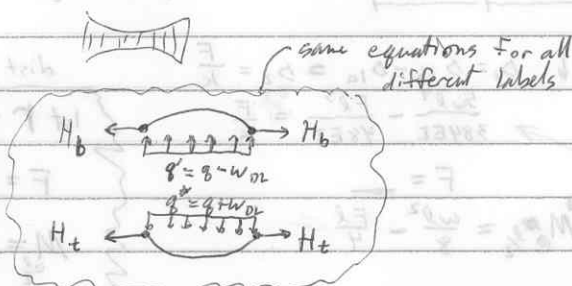
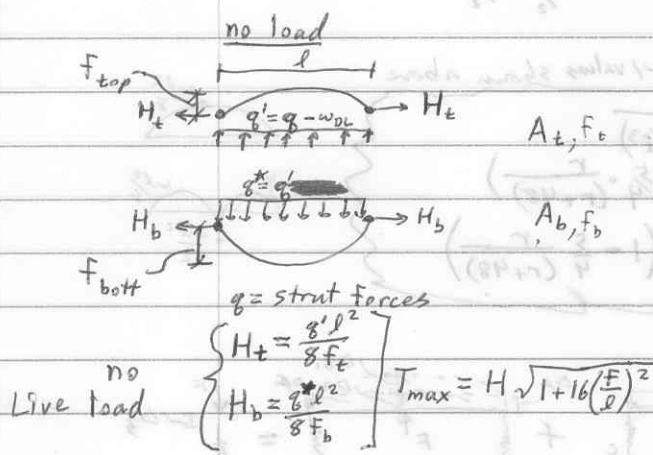
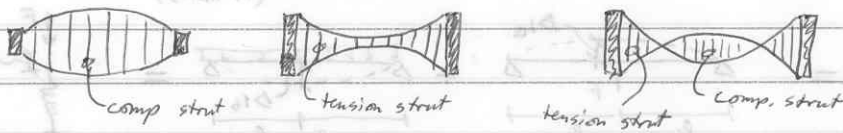
if  $F_{TOT} \neq F_{initial}$  (unloaded)

$T_{max} = \sqrt{(V)^2 + (H)^2}$

| cycle | starting F    | $\Delta F$        | ending F                         | conservative? ( $F_{end} > F_{start}$ ) | $F_{end} = F_{start}$ ?                       |
|-------|---------------|-------------------|----------------------------------|-----------------------------------------|-----------------------------------------------|
| 1     | $F_0 = 20'$   | $\Delta F = 2.81$ | $F_0 + 2.81 = 20 + 2.81 = 22.81$ | yes, I could stop                       | no                                            |
| 2     | $F_1 = 22.81$ | $\Delta F = 2.19$ | $F_0 + 2.19 = 20 + 2.19 = 22.19$ | no                                      | closer                                        |
| 3     | $F_2 = 22.19$ | $\Delta F = 2.31$ | $F_0 + 2.31 = 20 + 2.31 = 22.31$ | yes, I could stop                       | have found $F_{actual} = F_{start} = F_{end}$ |

# Combined tension structures

- assumptions
1. Both cables parabolic shape
  2. struts are continuous diagonals
  3. struts exert uniform load



add loading (LL)

$$\Delta q = w_{LL} \left( \frac{A_b f_b^2}{A_t f_t^2 + A_b f_b^2} \right)$$

$$\Delta H_t = \frac{(q' - \Delta q) l^2}{8 f_t}$$

$$\Delta H_b = \frac{(\Delta q) l^2}{8 f_b}$$

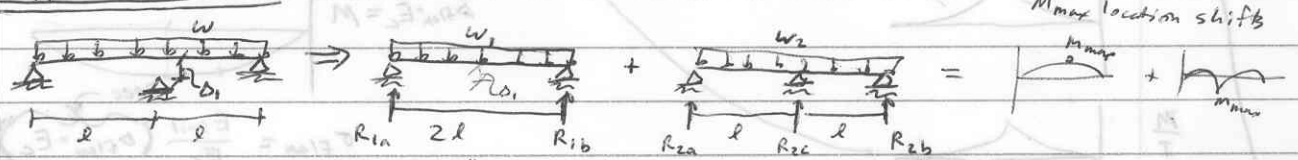
$$\Delta T = \Delta H \sqrt{1 + 16 \left(\frac{f}{l}\right)^2}$$

$$\bar{H}_t = H_t - \Delta H_t$$

$$\bar{H}_b = H_b + \Delta H_b$$

$$\bar{T}_{max} = \bar{H} \sqrt{1 + 16 \left(\frac{f}{l}\right)^2} = \begin{cases} T_t = 40k + 20.6k \\ T_b = T_{max} - \Delta T \end{cases}$$

## Support Settlement ( $w = w_1 + w_2$ )



$$\Delta_1 = \frac{5 w_1 l^4}{384 EI}$$

solve for  $w_1$   
solve for reactions

$$w_2 = w - w_1$$

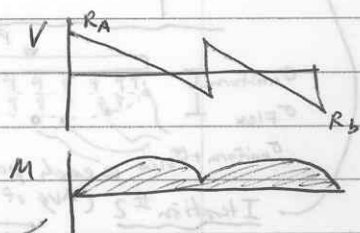
solve for reactions

add shears

$$R_A = R_{1A} + R_{2A}$$

$$R_C = R_{1C} + R_{2C}$$

$$R_B = R_{1B} + R_{2B}$$



compare w/ beam w/o settlement issue