IE 302
Engineering Economy

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Class 11

Rate of Return

Many different terms are used to refer to **rate of return**, including **yield** (e.g., yield to maturity, commonly used in bond valuation), **internal rate of return**, and **marginal efficiency of capital**. Three common definitions of rate of return will be reviewed first and then the definition of internal rate of return as a measure of profitability for a single investment project will be used throughout this class.

Return on Investment

There are several ways of defining the concept of rate of return on investment. Two of them are as follows:

- The first is based on a typical transaction, and
- The second is based on the mathematical expression of the present-worth function.

Definition 1 *Rate of return is the interest earned on the unpaid balance of an amortized loan*.

Suppose that a bank lends $10,000, which is repaid in installments of $4,021 at the end of each year for three years. How would the interest rate that the bank charges on this transaction be determined ? As discussed earlier, i   is obtained by solving the following equation :

• It turns out that i = 10%. In this situation, the bank will earn a return of 10% on its investment of $10,000. The bank calculates the loan balances over the life of the loan as follows

Year	Unpaid balance at the beginning of year	Return on unpaid (10%)	Payment Received	Unpaid balance at end of year
0				
1				
2				
3				

• A negative balance indicates an unpaid balance.

- It can be seen from the table on the previous page that, the 10% interest is calculated only for each year’s outstanding balance.
- In this situation, only part of the $4,021 annual payment represents interest; the remainder goes toward repaying the principal. In other words, the three annual payments repay the loan itself and provide a return of 10% on the amount still outstanding each year.
- Also, when the last payment is made, the outstanding principal is eventually reduced to zero.

- The NPW of the loan transaction at its rate of return (10%), is zero as seen below.

PW(10%) =

which indicates that the bank can break even at a 10% rate of interest.

- In other words, the rate of return becomes the rate of interest

• This observation prompts the second definition of rate of return:

Definition 2 *Rate of return is the break-even interest rate i* at which the net present worth of a project is zero, or*

PW(i*) =

The foregoing expression is equivalent to

- Here, the value of A_n for each period is known, but not the value of i*. Since it is the only unknown and it can be solved for.

- This solution is not always straightforward due to the nature of the PW function in the above equation ; it is certainly possible to have more than one rate of return for certain types of cash flows.

• As the Future Worth and Annual Worth values are related to the Present Worth value through non-negative numbers, the i* of a project may also be defined as

Return on Invested Capital

Investment projects can be viewed as analogous to bank loans. The concept of rate of return based on the return on invested capital in terms of a project investment will be introduced now. A project's return is referred to as the internal rate of return (IRR), or the **yield** promised by an **investment project** over its **useful life**.

Definition 3 *The internal rate of return is the interest rate charged on the unrecovered project balance of the investment such that, when the project terminates, the unrecovered project balance is zero.*

Suppose a company invests $10,000 in a computer with a three-year useful life and equivalent annual labor savings of $4,021. Here, the investing firm may be viewed as the lender and the projects as the borrower. The cash flow transaction between them would be identical to the amortized loan transaction described under Definition 1:

Year	Beginning Project Balance	Return on Invested Capital (10%)	Cash Generated from Project	Project Balance at end of year
0				-10,000
1	-10,000	-1,000	4,021	-6,979
2	-6,979	-698	4,021	-3,656
3	-3,656	-366	4,021	0

• In the project-balance calculation, it can be seen that 10% is earned (or charged) on $10,000 during year one, 10% is earned on $6,979 during year two, and 10% is earned on $3,656 during year three.

• This information indicates that the firm earns a 10% rate of return on funds that remain *internally* invested in the project. Since it is a return *internal* to the project, this is referred to it as the **internal rate of return**, or IRR.

- This means that the computer project under consideration brings in enough cash to pay for itself in three years and to provide the firm with a return of 10% on its invested capital.

- To put it differently, if the computer is financed with funds costing 10% annually, the cash generated by the investment will be exactly sufficient to repay the principal and the annual interest charge on the fund in three years.

• Only one cash outflow occurs in year 0, and the present worth of this outflow is simply $10,000.

•There are three equal receipts, and the present worth of these inflows is

• Since PW =

10% also satisfies Definition 2 for rate of return.

• Even though this simple example implies that i* coincides with IRR, only Definitions 1 and 3 correctly describe the true meaning of internal rate of return.

- It will be seen later that if the cash expenditures of an investment are not restricted to the initial period, several break-even interest rates may exist that satisfy the equation

- However, there may not be a rate of return *internal* to the project.

Methods for Finding Rate of Return

The value of i^* can be found by several procedures, each of which has its advantages and disadvantages. The types of investment cash flow will be classified next to facilitate the process of finding the rate of return for an investment project.

Simple versus Nonsimple Investments

- An investment project can be classified by counting the number of sign changes in its net cash-flow sequence.
- A change from either “-“ to “+” or “+” to “-“ is counted as one sign change. (A zero cash flow is ignored.) Then the following categories can be established:

- **A simple (or conventional) investment** is an investment in which the initial cash flows are negative and only one sign change occurs in the net cash flow series. If the initial flows are positive and only one sign change occurs in the subsequent net cash flows, the flows are referred to as **simple-borrowing** cash flows.

- A **nonsimple (or nonconventional) investment** is an investment in which more than one sign change occurs in the cash flow series.

• Multiple i*s, occur only in nonsimple investments. If there is no sign change in the entire cash flow series, no rate of return exists. The different types of investment possibilities may be illustrated as follows:

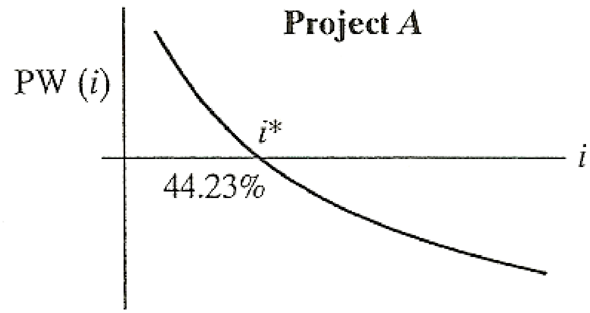
Investment Type	Cash Flow Sign at Period						The number of sign changes
	0	1	2	3	4	5	
Simple	-	+	+	+	+	+	
Simple	-	-	+	+	0	+	
Nonsimple	-	+	-	+	+	-	
Nonsimple	-	+	+	-	0	+	

Example : Investment Classification

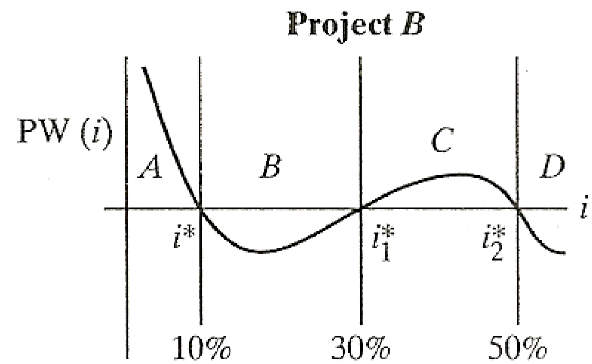
Classify the following three cash flows series as either simple or nonsimple investments:

	Net Cash Flow		
Period	Project A	Project B	Project C
0	-1,000	-1,000	1,000
1	-500	3,900	-450
2	800	-5,300	-450
3	1,500	-2,145	-450
4	2,000		

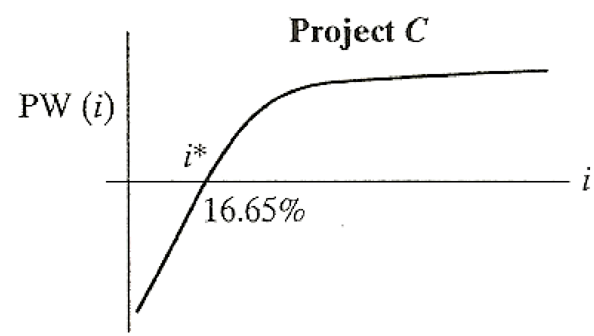
The figure plots the present worth of each project, PW (i) , as a function of the interest rate, i. The value of i which makes PW (i) = 0, is i*.



(a)



(b)



(c)

SOLUTION

- Project A represents many common simple investments. The curve crosses the i-axis only once.
- Project B represents a nonsimple investment. The i-axis is crossed at 10%, 30%, and 50%.
- Project C represents neither a simple nor a nonsimple investment, even though only one sign change occurs in the cash-flow sequence. Since the first cash flow is positive, this flow is a simple-borrowing cash flow, not an investment flow.

Computational Methods

Once the type of an investment cash flow is identified, there are several ways to determine its rate of return. Some of the most practical methods will be discussed next. They are as follows:

- Direct-Solution Method
- Trial-and error method

Direct-Solution Method

For the very special case of a project with only a two-flow transaction (an investment followed by a single future payment) or a project with a service life of two years of return, a direct analytical solution method for determining the rate of return can be used. These two cases are examined in the next example.

Example

Consider two investment projects with the following cash-flow transactions:

N	Project A	Project B
0	-3,000	-2,000
1	0	1,300
2	0	1,300
3	0	
4	4,500	

Compute the rate of return for each project.

SOLUTION

Project 1: Solving for i* in PW(i*) = 0 is identical to solving for i* in FW(i*) = 0, because FW equals PW times a constant. Either method could be used here, but FW(i*) = 0 is used. Using the single-payment future-worth relationship,

FW(i) =

Solving for i yields

$$i^* =$$

Project 2: The PW expression can be written for this project as follows:

PW (i) =

The PW(i) expression can be rewritten as a function of X and set it equal to zero, as follows

$$PW(i) =$$

This expression is a quadratic equation that has the following solution:

$$X =$$

Replacing the X values and solving for i gives us

And

Since an interest rate less than -100% has no economic significance, the project's i* is 25%.

- In both projects, one sign change occurred in the net cash-flow series and hence there was a unique i*. Also, these projects had very simple cash flows. Generally, when cash flows are more complex, a trial-and-error method or a computer program is used to find i*.

Trial-and-Error Method

The first step in the trial-and-error is to make an estimated guess at the value of i*. For a simple investment, this guessed interest rate is used to compute the present worth of net cash flows and whether the result is positive, negative, or zero is noted:

- **Case 1**: PW(i) < 0.

- **Case 2**: PW(i) > 0.

- When PW(i) is bounded by one negative value and one positive value, then **linear interpolation** can be used to approximate i*. This process is somewhat tedious and inefficient. (The trial-and-error method does not work for nonsimple investments in which the PW function is not, in general, a monotonically decreasing function of interest rate.)

Example : Finding i* by Trial and Error

ACME Corporation is considering a proposal to establish a facility to manufacture an electronically controlled “intelligent” crop sprayer . This crop sprayer project would require an investment of $10 million in assets and would produce an annual after-tax net benefit of $1.8 million over a service life of eight years. When the project terminates, the net proceeds from the sale of the assets would be $1 million. Compute the rate of return of this project.

SOLUTION: Let the initial guessed value of the interest rate be 8%. The net present worth of the cash flows in millions of dollars is

PW(8%) =

- Since this net present worth is positive, the interest rate must be increased in order to bring the present worth value toward zero. Let the new interest rate be 12%, which yields

  PW(12%) =

Now the solution is bounded between two values. PW(i) will be zero at i somewhere between 8% and 12%. Using straight-line interpolation, i* is obtained as

- Now it is necessary to check how close this value is to the precise value of i*. The net present worth at this interpolated value is

PW(10.30%) =

As this result is not zero, the present worth is recomputed at a lower interest rate, say, 10%:

PW(10%) =

With another round of linear interpolation, i* is obtained as

At this interest rate,

PW(10.18%) =

which is practically zero, and hence the search procedure stops here.

- In fact, there is no need to be more precise about these interpolations, because the final result can be no more accurate than the basic data, which ordinarily are only rough estimates. Incidentally, computing the i* for this problem on a computer yields a value of 10.1819%.

Example : Yield to Maturity

A person is considering buying a $1000-denomination corporate bond at the market price of $996.25. The interest will be paid semiannually at an interest rate per payment period of 4.8125%. This means that the owner of the bond will receive twenty semi-annual payments of size = 1000 * 0.048125 = $ 48.13 each, with the first payment at the end of six months from now (over a period of ten years).

At the end of the ten year period, the owner will receive an amount equal to the face value of the bond, which is $ 1,000. Find the return on this bond investment (or yield to maturity).

SOLUTION

- The yield to maturity is obtained by determining the interest rate that makes the present worth of the receipts equal to the market price of the bond:

- The value of i that makes the present worth of the receipts equal to $996.25 lies between 4.5% and 5%. Solving for i by interpolation yields i = 4.84%.

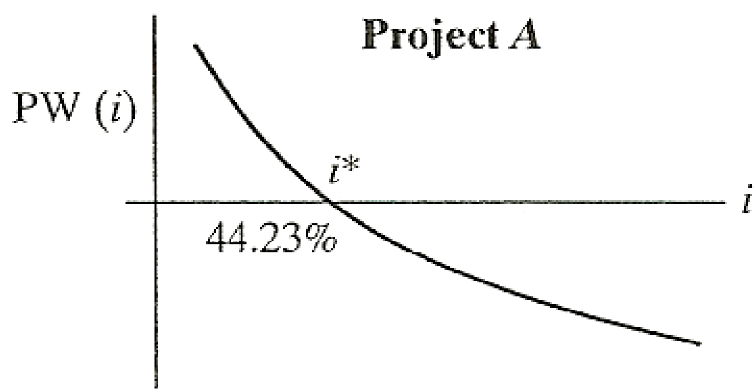
Internal-Rate-of-Return Criterion

- Now, an "accept-or-reject" decision rule will be developed that gives results consistent with those obtained from PW analysis.

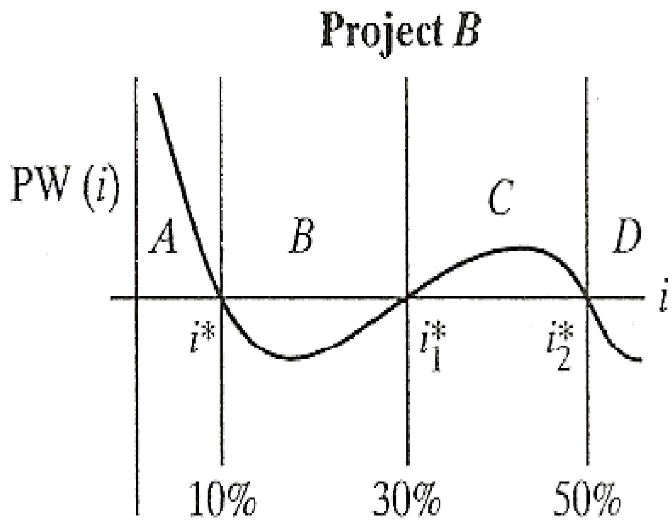
Relationship to the PW Analysis

- As discussed earlier, PW analysis is dependent on the rate of interest used for the PW computation. A different rate may change a project from being considered acceptable to being considered unacceptable, or it may change the ranking of several projects.

- Consider again the PW profile as drawn for the simple project A discussed earlier and reproduced below. For interest rates below i*, this project should be accepted, as PW > 0; for interest rates above i*, it should be rejected.



• On the other hand, for certain nonsimple projects, the PW may look like the one shown in the figure reproduced below for project B, discussed earlier.



- Use of PW analysis would lead one to accept the projects in regions A and C, but reject those in regions B and D.

- Of course, this result goes against intuition: A higher interest rate would change an unacceptable project into an acceptable one. This situation graphed in the figure is one of the cases of the multiple i*’s mentioned regarding Definition 2.

- Therefore, for the simple investment situations such as Project A, the i* can serve as an appropriate index for either accepting or rejecting the investment.

- However, for the nonsimple investments such as Project B, it is not clear which i* to use in order to make an accept-or-reject decision. Therefore, the i* value fails to provide an appropriate measure of profitability for an investment project with multiple rates of return.